Fourier for Power Signals

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1 Why frequency domain analysis?

Traditional power system analysis works on the assumption of pure sinusoidal waves representing currents and voltages through linear loads. However, given the growing ubiquity of consumer electronics like switched-mode power supplies that drive millions of personal computers, it is naïve to not consider the impact of non-linear loads on the power grid. The resulting waveform distortion is usually harmonic in nature; harmonics (mostly odd) of the fundamental frequency are present in varying amounts.

The substantial peak-to-average power ratio in current that is created by such non-linear devices has undesirable operational effects on transmission and distribution system components, and these effects need to be studied. Various power quality metrics have been defined to quantify the impact of waveform distortion, like Total Harmonic Distortion (for power being delivered by the grid at harmonic frequencies) and K-Factor ratings (for effects on transformer core heating). In these scenarios, the sinusoidal models of analysis quickly become inadequate – constructs like VARs and power factor no longer have any meaning.

2 Frequency Domain - The Fourier Transform

The Discrete Fourier Transform is a linear mathematical operator which breaks a finite discrete time sequence into a sum of harmonic components that are evenly spaced in frequency. For a sequence $x[n]$ with N samples, the DFT is calculated as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}, \; k = 0,\, 1,\, 2 \ldots N-1 \tag{1}$$

This is just the Discrete Time Fourier Transform (DTFT) equation calculated at the following equally spaced frequency points, where F_S is the sampling rate:

$$\frac{2\pi}{F_S}\left\{0,\, \frac{F_S}{N},\, \frac{2F_S}{N} \ldots \frac{(N-1)F_S}{N}\right\}$$

Recall that the DTFT for a sequence $x[n]$ is:

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x[n]e^{-j\omega n} \tag{2}$$

For real signals, the DFT is a complex function of frequency that has two important properties.

1. The function is periodic about the sampling rate. This sampling rate must be at least twice the frequency of the highest frequency component of the (band-limited) signal for aliasing to be avoided.
2. The function's amplitude spectrum is even.

The brute force method of evaluating the DFT function at each frequency point leads to a number of calculations proportional to N^2. A more efficient way of calculating a DFT involves decomposing the computation into successively smaller DFT computations and exploiting the properties listed above. This is the FAST FOURIER TRANSFORM (FFT). Computer-based implementations of the FFT lead to the further requirement of having N be a power of two. The number of complex additions and multiplications now are of the order of $N \log_2(N)$.

3 Restrictions of FFT

Even though the FFT routines are computationally efficient, they are not suited for all situations. For ADE7878-based applications where the source of digital data is the ADE7878, the FFT is particularly inappropriate.

1. Spectral Over-sampling: The FFT calculates the transform at frequency points up to the sampling rate, which is 8 KHz for ADE7878. However, there is a digital low pass filter inside the ADE7878 with a cut-off at 2 KHz. The frequency content of the analog input above 2 KHz is therefore not reflected in the digital output. This renders the calculation of the response above 2 KHz unnecessarily time consuming.
2. Non-Integral Sampling: The nominal system frequency of power systems is either 50 or 60 Hz, and any waveform distortion is usually harmonically related to the fundamental. It is then desirable to have the frequency points of the transform hit these fundamental and harmonic frequencies. As mentioned before, computer-based implementations of the FFT require the number of frequency points where the transform is to be calculated to be a power of two, i.e. $N = 2^k$. It becomes challenging to set a value of k such that

$$i = \begin{cases} \frac{50 \bullet 2^k}{8000} & for\ 50\ Hz \\ \frac{60 \bullet 2^k}{8000} & for\ 60\ Hz \end{cases} \tag{3}$$

is an integer.

3. Base Frequency Slippage: Based on the operating conditions of the power grid, the system base frequency may not be a stiff value. It is normal for the frequency to vary around the base by up to 1%. In fact, the IEC 61000 specifications mandate that a metrology system should be able to handle base frequency variations of up to 1 Hz. The FFT routine, once set, would not be capable of altering its frequency constellation to track the changing base frequency. While it is not impossible to set up a scalable FFT system (such as those implemented in WiMax modems as a part of S-OFDMA), the computation is heavily time and resource consuming.

4 DTFT Revisited

The above described limitations of the FFT lead to a reformulation of the basic Discrete Time Fourier Transform problem in terms of polynomial decomposition. Note that the DTFT equation is in fact a polynomial of order *N-1,* with real coefficients and a complex argument $z_0 = e^{-j\omega_0} = x_0 + jy_0$.

A general N^{th} order polynomial with a complex argument is given by

$$p(z) = a_0 + a_1 z + a_2 z^2 + \dots a_n z^n \tag{4}$$

The a coefficients here represent the discrete time series $x[n]$ (see equation 2). This can be re-written as:

$$p(z) = b_0 + b_1(z - 2x_0) + (z - z_0)(z - z_0^*)(b_n z^{n-2} + b_{n-1} z^{n-3} + \dots b_3 z + b_2) \tag{5}$$

This simplifies to

$$p(z) = \begin{cases} \left[b_0 - b_1 2x_0 + b_2(x_0^2 + y_0^2)\right] & + \\ z\left[b_1 - b_2 2x_0 + b_3(x_0^2 + y_0^2)\right] & + \\ z^2\left[b_2 - b_3 2x_0 + b_4(x_0^2 + y_0^2)\right] & + \\ \dots & \dots \\ z^{n-1}\left[b_{n-1} - b_n 2x_0\right] & + \\ z^n b_n \end{cases} \tag{6}$$

Note that the following one-to-one correspondence holds:

$$\begin{aligned} b_0 - b_1 2x_0 + b_2(x_0^2 + y_0^2) &= x[0] \\ b_1 - b_2 2x_0 + b_3(x_0^2 + y_0^2) &= x[1] \\ b_2 - b_3 2x_0 + b_4(x_0^2 + y_0^2) &= x[2] \\ &\dots \end{aligned}$$

The polynomial, when evaluated at z_0, reduces to

$$p(z_0) = b_0 - b_1 x_0 + j b_1 y_0 \tag{7}$$

To get the coefficients b_0 and b_1, note that equation 6 is in the shape of an IIR filter. To see this, reverse the time indecies of both the b coefficients and of the discrete sequence (whose transform is to be calculated). If the new sequences are represented as $\{l_0, l_1, \ldots l_n\}$ and $\{C_0, C_1, \ldots C_n\}$ respectively, where $l_0 = b_n$, and $C_0 = x[n]$, the resulting IIR filter has the following transfer fuction:

$$H(z) = \frac{1}{1 - 2x_0 z^{-1} + (x_0^2 + y_0^2) z^{-2}} \tag{8}$$

The coefficients b_0 and b_1 are obtained as the last two elements of the output of this filter when the input is the time reversed discrete sequence $x[n]$.

5 Some Implementation Details

To get the spectral decomposition of an input sequence $x[n]$, the Fourier Transform has to be calculated, as described above. For this, the frequency constellation at which the transform has to calculated needs to be known. For the ADE7878, the two constraints in this regard are that the grid frequency can change with time, and that there is a 2 KHz low pass filter[1] in the ADE7878.

To track the base frequency, the PERIOD register of ADE7878 is polled regularly. This register is associated with a set of sophisticated zero-crossing detectors inside the chip that continually measure the time between two successive zero crossings of the Phase A voltage wave. To avoid any discrepancy that can arise due to, for example, multiple zero crossings of a harmonically laden wave with in a single time period, the chip passes the waveform through a low pass filter in its ZXD signal chain[2]. The frequency readings from the PERIOD register are passed through a smoothing filter. This filter has been designed to be able to track small variations of the base frequency without noticeable delay, but not capture very sudden changes such as those associated with an MV fault event. In a fault event, a decaying DC component may get introduced to the waveform that may (erroneously) get reflected in the instantaneous value of the Period register as a reduced frequency.

Once the base frequency (fundamental) is obtained, the frequency constellation vector is calculate as:

$$\{F_o, 2F_o, 3F_o, \ldots kF_o\}$$

where k is such that

$$kF_o \leq 1500\ Hz$$

[1]The filter response begins to dip down at 1500 KHz

[2]This filter is different from the 2 KHz low pass filter mentioned earlier